Using Combinatorial Benchmarks to Probe the Reasoning Power of Pseudo-Boolean Solvers

Jakob Nordström

KTH Royal Institute of Technology
Stockholm, Sweden

Pragmatics of Constraint Reasoning
Melbourne, Australia
August 28, 2017

Joint work with Jan Elffers, Jesús Giráldez-Cru, and Marc Vinyals

Or: A Tale of Four Formulas...

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Focus:

- Applied pseudo-Boolean solving
- Proof complexity of cutting planes
- Connections between the two (or not)

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Experimental evaluations of:

- Sat4j [S4j, LP10]
- cdcl-cuttingplanes [Elf16] (*cdcl-CP* for short)
- Open-WBO [Ope, MML14]

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- Open-WBO [Ope, MML14]

Open-WBO:	Re-encoding to CNF + CDCL
Sat4j & cdcl-CP:	Conflict-driven search natively with PB constraints

Pigeonhole Principle Formula

$$\sum_{j=1}^{n} x_{i,j} \geq 1 \qquad\qquad i \in [n+1]$$

$$\sum_{i=1}^{n+1} x_{i,j} \leq 1 \qquad\qquad j \in [n]$$

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How to show unsatisfiable?

- Sum up all pigeons
- Sum up all holes
- Subtract to get $0 \geq 1$

Subset Cardinality Formula [Spe10, VS10, MN14]

Variables = 1s in matrix with four 1s per row/column + extra 1
Each row wants majority true; each column wants majority false

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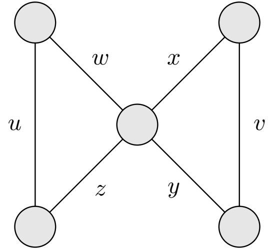
How to show unsatisfiable?

- Sum up greater-equal constraints for rows
- Sum up less-equal constraints for columns
- Subtract to get $0 \geq 1$

Even Colouring Formula [Mar06]

$G = (V, E)$ connected graph; all $\deg(v)$ even
Constraints $\sum_{e \ni v} x_e = \deg(v)/2$



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Inconsistent iff $|E|$ odd

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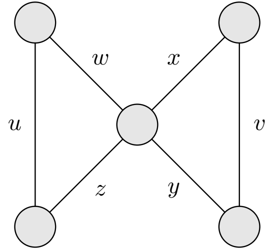

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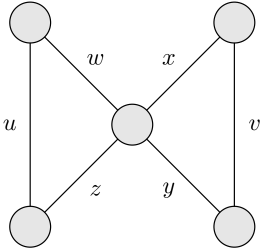

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- Sum up less-equal constraints, divide, and round down
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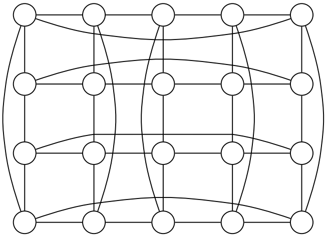
Vertex Cover Formula [VEG+17]



Graph $G = (V, E)$, size $S \in \mathbb{N}^+$

$$\sum_{v \in V} x_v \leq S$$

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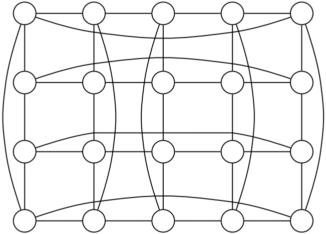

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Take $m \times n$ rectangular, toroidal grid; m even; n odd
Inconsistent for $S = mn/2$ (or even $S = m\lceil n/2 \rceil - 1$)

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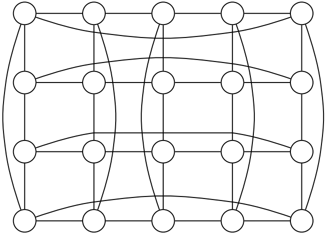

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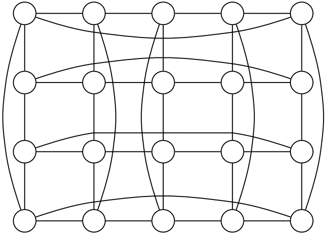

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How to show unsatisfiable?

- Sum over edges in each row, divide, and round up
- Subtract size constraint to get $0 \geq 1$

Theory vs. Practice

All these instances supereasy in theory (tree-like cutting planes)

Theory vs. Practice

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What about practice?

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Investigate "same instance" scaled to different sizes
Study asymptotic behaviour (no cactus plots)

- **Pigeonhole principle**
  Super-easy for cdcl-CP & Sat4j; dead-hard for Open-WBO
- **Subset cardinality**
  Super-easy for cdcl-CP & Sat4j; dead-hard for Open-WBO
- **Even colouring**
  Challenging but doable for cdcl-CP & Sat4j (though depends on graph)
  Hard for Open-WBO (though depends *a lot* on graph)
- **Vertex cover**
  Very challenging for cdcl-CP & Sat4j; super-easy for Open-WBO

How to Explain This?

- Rational v.s. Boolean solutions?
- Pseudo-Boolean proof search and backdoors?
- Pseudo-Boolean solving vs. CDCL?

Rational v.s. Boolean Solutions?

Observation:

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- More challenging when $\exists$ rational but not Boolean solutions

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Rational Hypothesis

Pseudo-Boolean solver performance correlates with rational unsatisfiability

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- More challenging when $\exists$ rational but not Boolean solutions

Rational Hypothesis

Pseudo-Boolean solver performance correlates with rational unsatisfiability

- Beautiful hypothesis (or at least I thought so)
- Only one problem: Not backed up by data

Pseudo-Boolean Proof Search and Backdoors?

More detailed observation about cdcl-CP & Sat4j:

- Can make run fast when $\exists$ small backdoors to no rational solutions
- By tweaking heuristics, but not changing proof search fundamentals

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Pseudo-Boolean solvers have potential to run fast when there are small, strong backdoors to rational unsatisfiability

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Extended Rational Hypothesis

Pseudo-Boolean solvers have potential to run fast when there are small, strong backdoors to rational unsatisfiability

- Clearly not if-and-only-if — instances can be easy for other reasons
- If-direction true in theory even for weakest PB proof system
- Seems to hold in practice for (almost) all instances we have studied
- But this is still ongoing work
- What would the practical implications be? (Full division rule needed?)

Pseudo-Boolean Solving vs. CDCL?

- Instance hard for resolution $\Rightarrow$ Open-WBO has no chance

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- Even "easy versions of hard formulas" can be dead-hard (subset cardinality, even colouring)
- Open-WBO can be very good when cardinality encoding works well (vertex cover: 2-CNF $+$ big cardinality constraint)
- But very sensitive to input ordering — should we trust nicety of encodings or prefer robust solvers?

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Pseudo-Boolean Solving vs. CDCL?

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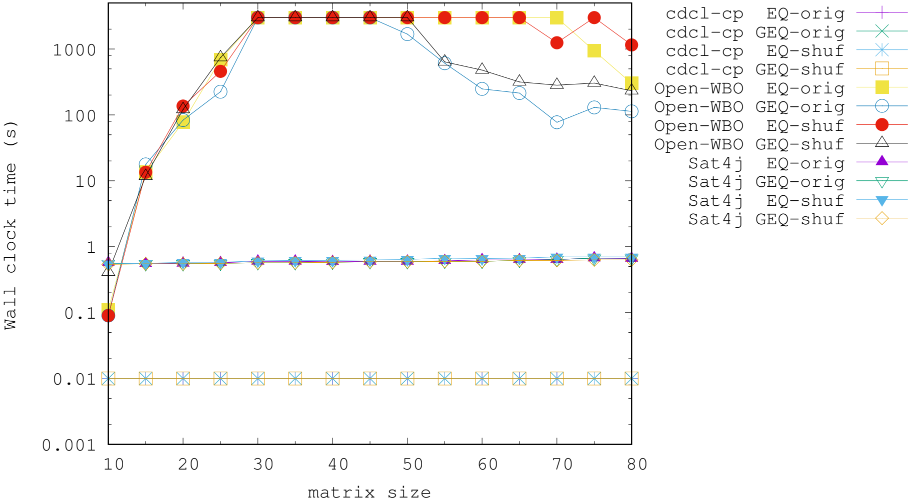
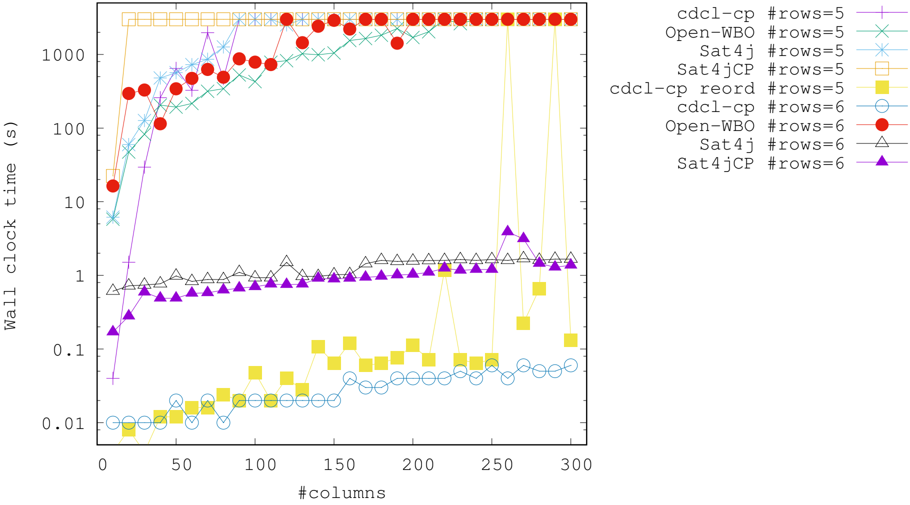
- But cdcl-CP deviates if given free choice — what makes Open-WBO stick with good order?

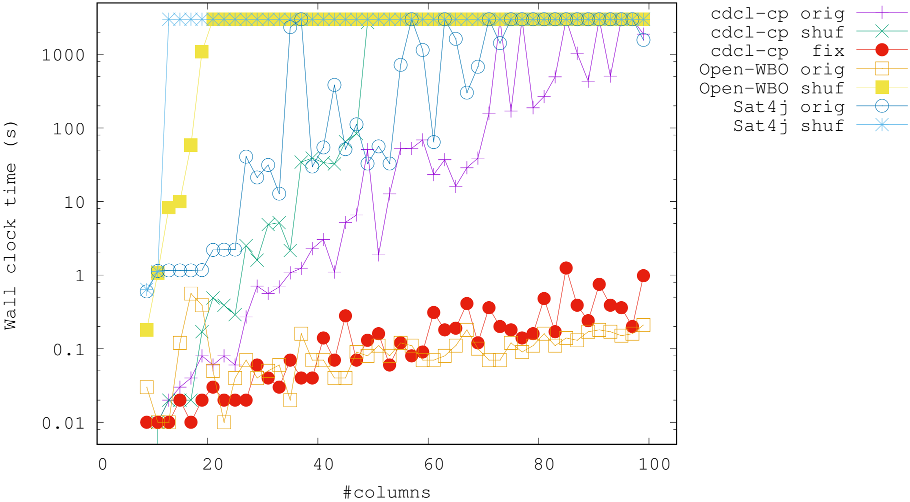
Subset Cardinality for Fixed Bandwidth Matrices



Even Colouring on Rectangular Grids



Vertex Cover on Grids (Rationally UNSAT)



Take-Home Messages

- Study easy, but tricky, crafted instances (not super-hard ones)
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- Talk to me or e-mail `jakobn@kth.se` if interested

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Thank you for your attention!

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