Visualizing CDCL VSIDS and phase values

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Visualizing CDCL

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- Variable decisions.
- Clause database management.
- Restarts.

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- Restarts.

Heuristics often work very well.

Limited understanding of **why**.

This presentation: tool for visualizing CDCL heuristics (so far on crafted benchmarks)

Visualizing CDCL: decision heuristic

Classic decision heuristic: variable with highest VSIDS score [MMZ$^+$01]

If variable v was active in conflicts $t_1, \ldots, t_k$ and T conflicts have passed,

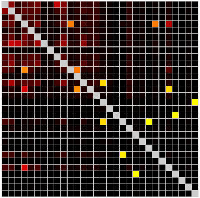
$$activity(v) = \sum_{1 \leq i \leq k} \alpha^{T - t_i}$$

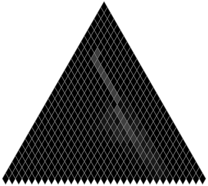
α decay factor $1/2 \leq \alpha < 1$. (default 0.95)

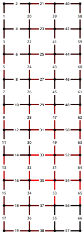
Phase saving [PD07]: Set variable to the polarity (phase) it was last propagated to.

Visualizing VSIDS scores

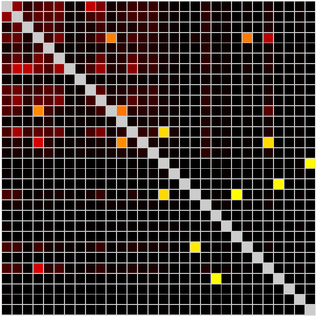
- Project arose out of understanding how CDCL solves tricky combinatorial benchmarks.
- By visualizing aspects of proof search (VSIDS, phase) we try to understand CDCL.
- Crafted benchmarks often defined in terms of graphs or matrices.







Visualizing VSIDS scores



Brighter color = higher score

- Runs during CDCL execution.
- Refreshes every 100 conflicts.*
- At most 25 "frames"/second.*

*numbers can be changed

How to implement your own visualization

Visualizer written in Qt framework.

- Before the solver starts running, a method is called to draw shapes on the grid.
  Each shape = a variable.
  In this method you can draw your own visualization.
- With each new frame, a method
  `draw_vsids(vector<double> scores)` is called.
  Gives shapes colors corresponding to VSIDS scores.
  Can be overridden to define custom color scheme.

Smoothing of VSIDS animation

CDCL VSIDS scores can change quickly: becomes a blur.

- Use exponential moving average on VSIDS scores:

$$\mathit{expAverage}(v)_t = \sum_{\Delta t \geq 0} \alpha^{\Delta t} \mathit{activity}(v)_{t-\Delta t}$$

  set α to say 0.99.

  $\mathit{activity}(v)_t$ are VSIDS scores (= exponential averages) themselves

- Visualizer reads VSIDS scores from solver.
- Visualize frames every 100 conflicts.
  However, want average over *all* conflicts
  Cannot read full VSIDS scores after every conflict.

Smoothing of VSIDS animation

Our implementation: let solver maintain 2 VSIDS scores: standard and more slowly decaying.

Mathematical guarantee:

Theorem

Exponential average with parameter d_{slow} over decay d VSIDS is a linear combination of decay d VSIDS and decay d_{slow} VSIDS.

$\Rightarrow$ slower decay factor approximates exponential moving average.

Phase VSIDS

Similar problem with phases. Can apply same VSIDS idea. For $0 < \alpha < 1$, one can define

$$\mathit{vsidsPhase}(v)_t = (1-\alpha)\mathit{phase}(v)_t + \alpha \cdot \mathit{vsidsPhase}(v)_{t-1}$$

where false $= -1$, true $= 1$.
Only for visualization purposes currently.

Integration in CDCL solver

Communication via text streams on stdin / stdout

- Add a command line parameter "interactive" to the solver.
- Add a method `interactive()` to the solver.
  Idea:
  1. read #conflicts to run
  2. run solver for this many conflicts
  3. output *frame*:
     - The secondary VSIDS scores, normalized to $[0, 1]$.
     - Phases mapped to interval $[-1.0, 1.0]$ ($-1 =$ false, $1 =$ true).

Demo

1. VSIDS:
   (a) Ordering principle
   (b) Flow formula
2. Phase:
   (a) Pebbling formula with XOR
   (b) Pigeonhole principle

Demo 1a: ordering principle

A formula with variables $x_{i,j}$ for each $1 \leq i \neq j \leq n$ corresponding to edges of a complete graph.
Super easy in theory.
Hard for CDCL if VSIDS decay factor too high (too close to 1).

Clause deletion causes VSIDS resets, not restarts.

Demo 1b: flow formula with high distortion

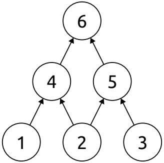
Formula defined on an undirected graph.
Super easy in theory.
Variables $x_{u,v}$ for all directed edges (u, v).
Constraints ($N(u)$ are neighbours of vertex u):

$$\forall u : \sum_{v \in N(u)} x_{u,v} - x_{v,u} = 1$$

Phenomenon observed: VSIDS “freezes” on some instances.
Sample graph is 6-regular random graph on 500 nodes.

Demo 2a: pebbling formula with XOR

A formula defined on a single-sink DAG with indegree 2.
Super easy in theory.



Two variables per vertex: $x_{u,1}$ and $x_{u,2}$. Constraints:

- $x_{u,1} \oplus x_{u,2} = 1$ for u a source.
- $x_{u,1} \oplus x_{u,2} = 0$ for u the sink.
- If u has predecessors v, w,
  $((x_{v,1} \oplus x_{v,2} = 1) \wedge (x_{w,1} \oplus x_{w,2} = 1)) \rightarrow (x_{u,1} \oplus x_{u,2} = 1)$

Visualize $phase(x_u) := phase(x_{u,1}) \cdot phase(x_{u,2})$ for all vertices u.
(remember $phase \in \{-1, 1\}$ without phase VSIDS)

Demo 2b: pigeonhole principle

Formula claims that n pigeons do not fit into $n-1$ holes.
Variables $x_{i,j}$ for $1 \leq i \leq n$, $1 \leq j \leq n-1$.
We run CDCL without restarts, and with and without phase VSIDS.

Conclusion

What is already implemented:

- Various visualizations for studying combinatorial formulas.
- Interactive version of Minisat for use with the visualizer.

What a user could add:

- Additional visualizations.
- Interactive versions of other CDCL SAT solvers.

The software is available on Github:
`github.com/elffersj/cdcl-visualizer`

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