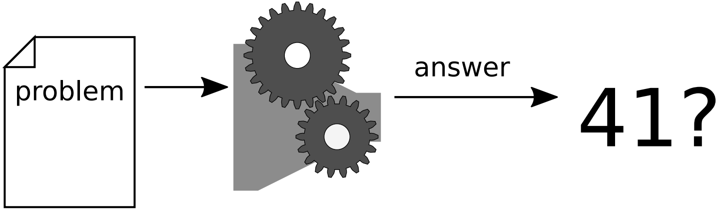
Certifying CNF Encodings of Pseudo-Boolean Constraints

Stephan Gocht, Ruben Martins and Jakob Nordström

July 2021

Detecting Bugs with Certifying Algorithms



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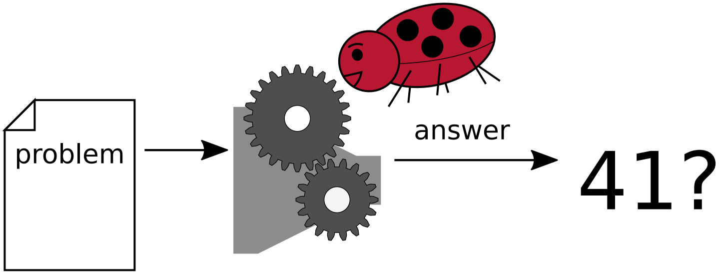

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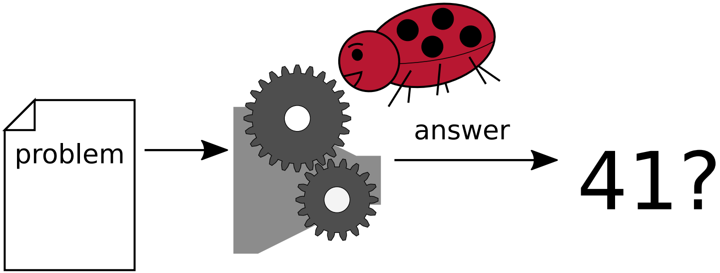

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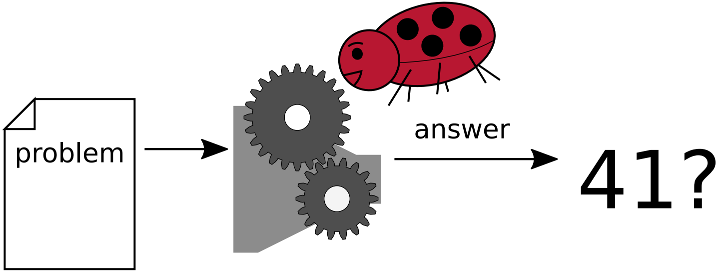

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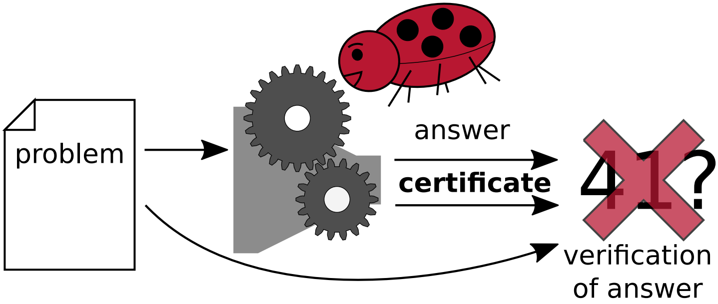

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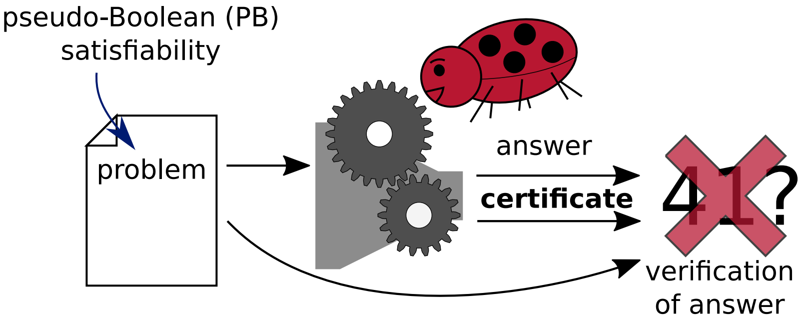

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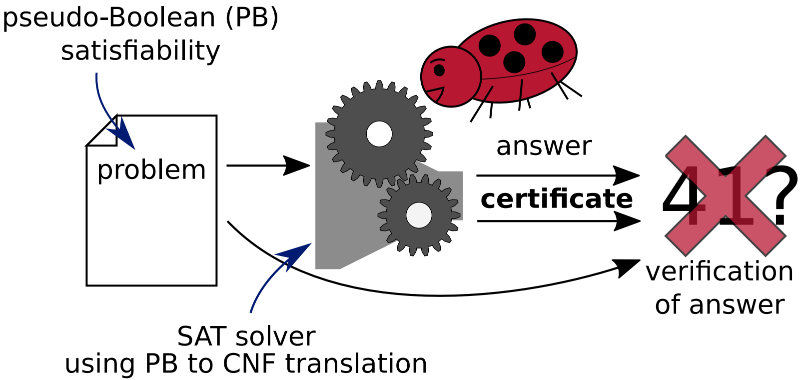

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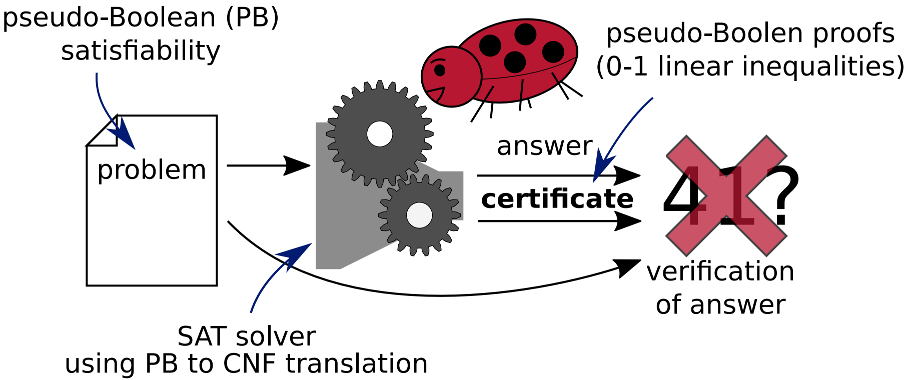

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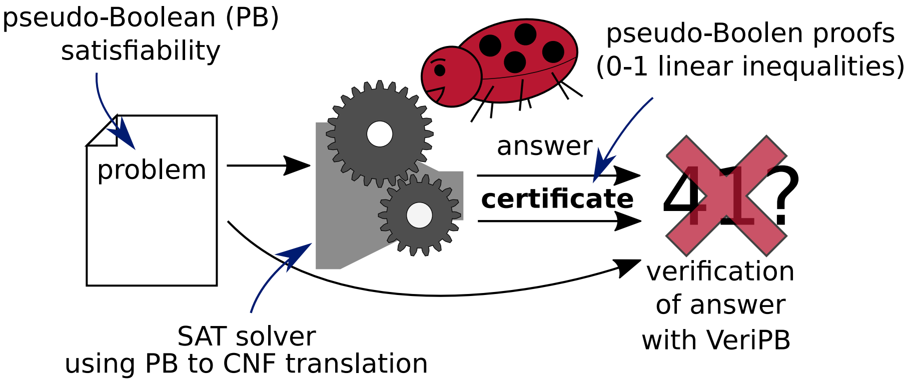

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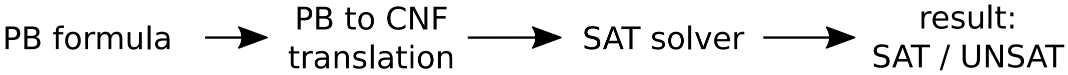
- certificates can help to
  - prove correctness of answer
  - detect and fix bugs, even when solver produced correct answer
  - audit answer later on
  - explain what solver is doing

. . . That is Easy to Use . . .



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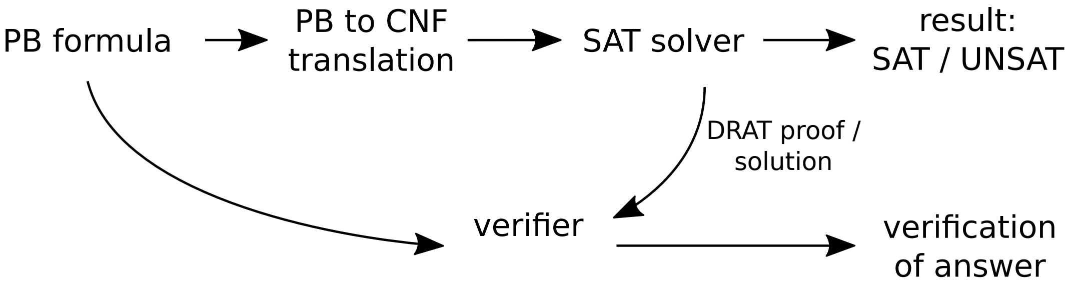

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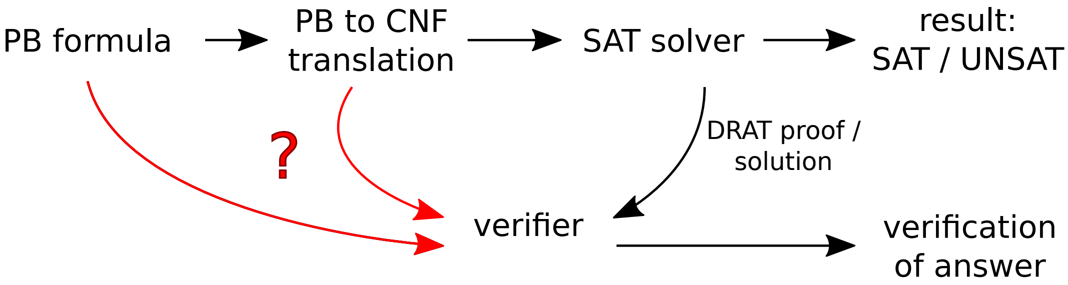

...Except for SAT Solving Techniques That Can't Be Certified

- too much overhead / too complicated proof logging for
  - Parity reasoning (as in CryptoMiniSat [Cry] and Lingeling [Lin])
  - Counting arguments (as in Lingeling)
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- How about practical proof logging for stronger solving paradigms?
  - MaxSAT solving
  - constraint programming (CP)
  - mixed integer programming (MIP)
  - algebraic reasoning / Gröbner basis computations
  - **pseudo-Boolean satisfiablity** and optimization

New Proof Systems on the Rise

many new proof systems with implemented proof checkers:

- propagation redundancy (PR) [HKB17]
- practical polynomial calculus (PAC) [RBK18, KFB20]
- propagation redundancy for BDDs [BB21]
- Max-SAT resolution [PCH21]
- **pseudo-Boolean proofs** [EGMN20, GN21]

Our Work

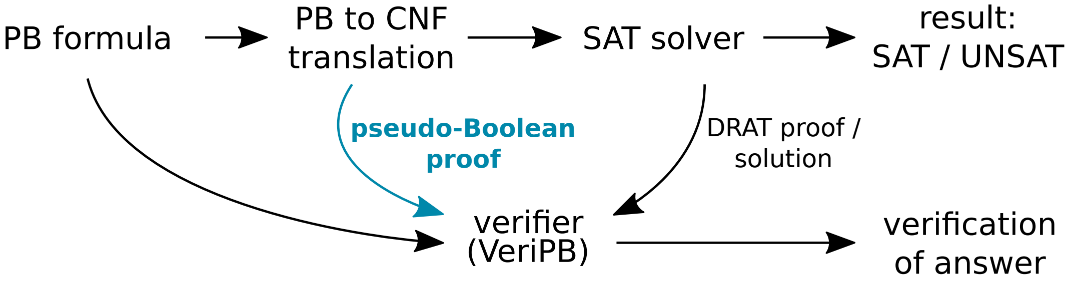
- general purpose proof format: pseudo-Boolean proofs (PBP)
- reference implementation of verifier: VeriPB[1]
- allows easy proof logging for
  - reasoning with 0-1 linear inequalities (by design)
  - all-different constraints [EGMN20]
  - subgraph isomorphism [GMN20]
  - clique and maximum common (connected) subgraph [GMM$^+$20]
  - parity reasoning [GN21]
  - SAT pre- and inprocessing [GN21]

This Work (by using VeriPB)

- proof logging for translating 0-1 linear inequalities to CNF (work in progress)
  so far only sequential counter [Sin05], many more encodings exist
- allows proof logging for SAT-based pseudo-Bolean solving

[1] `https://gitlab.com/MIAOresearch/VeriPB`

System Overview



Basic Notation

Starting from

$$x_1 + x_2 + x_3 \geq 2$$

want to derive

$$\bar{x}_1 + s_{1,1} \geq 1$$
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$$\bar{x}_2 + s_{2,1} \geq 1$$
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$$x_2 + s_{1,1} + \bar{s}_{2,1} \geq 1$$
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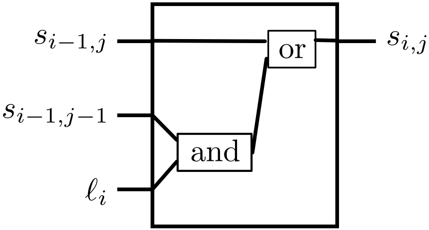
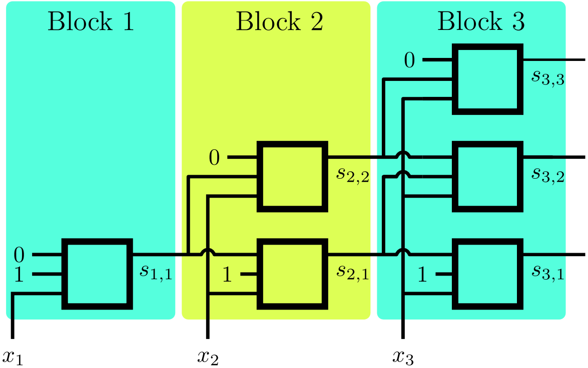
$$\dots$$

- Boolean variable x with domain 0 (false) or 1 (true)
- Literal: x or its negation $\bar{x} = 1 - x$
- Pseudo-Boolean (PB) constraint: linear (in-)equality over literals
- Clause: at-least-one constraint
- Proof Format:
  - based on pseudo-Boolean constraints
  - has operations to reason with PB constraints

Sequential Counter Encoding





High level specification:

Block 1: $x_1 = s_{1,1}$

Block 2: $x_2 + s_{1,1} = s_{2,1} + s_{2,2}$ $\quad s_{2,1} \geq s_{2,2}$

Block 3: $x_3 + s_{2,1} + s_{2,2} = s_{3,1} + s_{3,2} + s_{3,3}$ $\quad s_{3,1} \geq s_{3,2} \geq s_{3,3}$

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- Specification is pseudo-Boolean!
- $s_{i,j}$ variables are fresh, i.e., not in formula
- $\Rightarrow$ always OK to add these constraints
- can be derived in VeriPB proof system

From Specification to CNF

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$\Rightarrow$ derivation can be expressed in VeriPB proof system

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Our work: Proof logging translating 0-1 linear inequalities to CNF with VeriPB[2]

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Future work:

- provide efficient proof logging also for optimization (pseudo-Boolean optimization, MaxSAT, MIP)
- express MaxSAT techniques (e.g. core guided search, MaxHS) in PB language
- new expressive proof formats and verifiers for competitions (why not with VeriPB ;-))

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