Certified Core-Guided MaxSAT Solving

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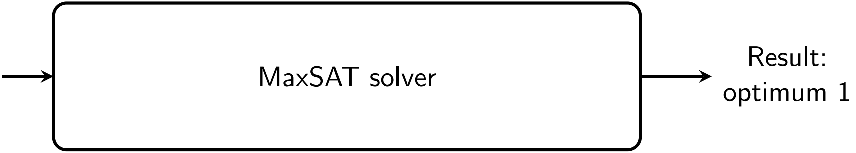
Joint CADE-29 paper with Jeremias Berg, Bart Bogaerts, Jakob Nordström and Dieter Vandesande

Maximum Satisfiability (MaxSAT) Solving

$$\begin{aligned} \min\ & 2x_1 + x_2 \\ \text{s.t. } & x_1 \vee \overline{z} \\ & z \vee x_2 \end{aligned}$$



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→ MaxSAT solver → Result: optimum 1

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- Equivalently: Maximize satisfied soft clauses subject to satisfying hard clauses

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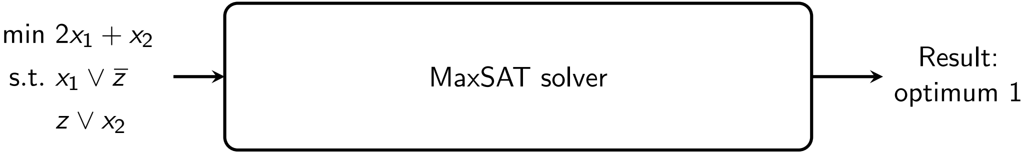

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- Main approaches:
  - Solution-improving or linear SAT-UNSAT search [ES06, LP10, PRB18]
  - Implicit hitting set (IHS) search [DB13a, DB13b]
  - **Core-guided search** [FM06, NB14, ADR15, AG17]

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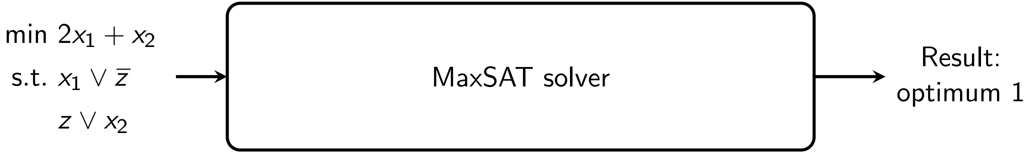

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How do we know if problem was solved correctly?

Correctness of Combinatorial Solvers

Testing:

- Can only show presence of bugs, not absence
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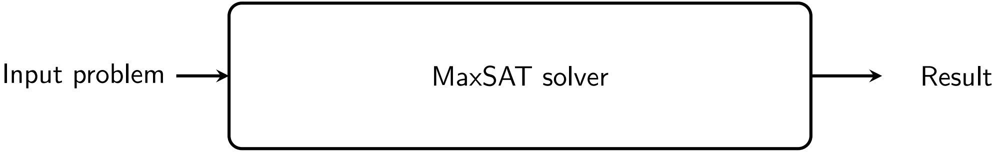
Proof logging (our approach):

- Guarantee that execution was correct
- Moderate overhead for implementing solver

Certifying Results with Proof Logging



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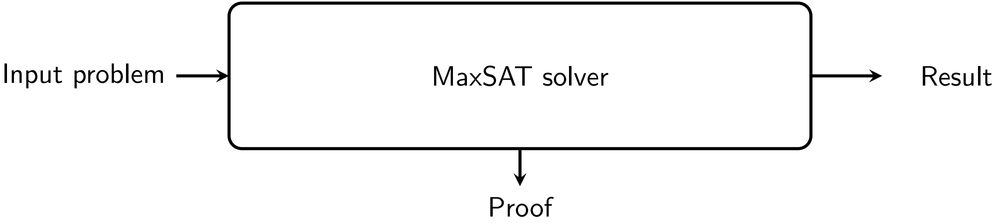

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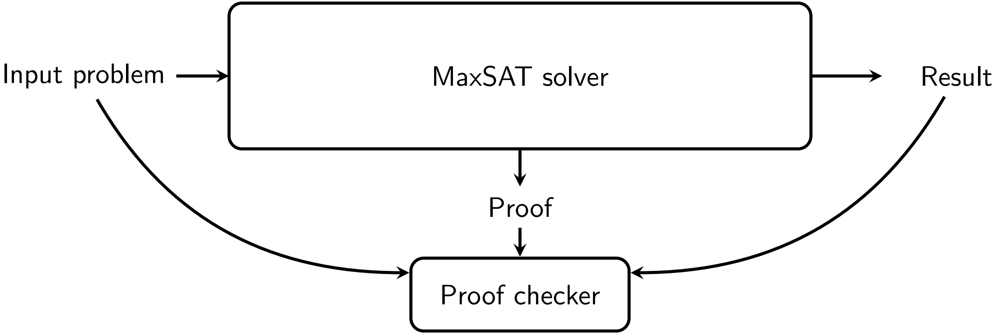

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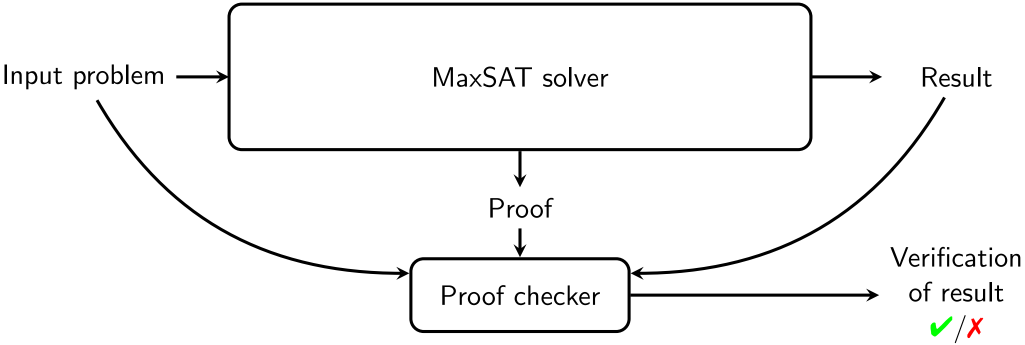

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Pseudo-Boolean (PB) Proof Logging

- **Multi-purpose** proof format
- Allows easy proof logging for
  - Reasoning with pseudo-Boolean constraints (by design)
  - SAT solving (including advanced techniques) [GN21, BGMN22]
  - Constraint programming [EGMN20, GMN22]
  - Subgraph problems [GMN20, GMM+20]
  - SAT-based pseudo-Boolean solving [GMNO22]
  - Unweighted linear SAT-UNSAT search MaxSAT [VDB22]

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This work:

- Proof logging for state-of-the-art core-guided MaxSAT solving [IMM19, IBJ21]

Rest of This Talk

1. Description of state-of-the-art core-guided MaxSAT solving
2. Our contribution: Adding proof logging to core-guided MaxSAT solving
3. Experimental evaluation
4. Conclusion

Basic Notation

- Boolean variable x: Domain 0 (false) and 1 (true)
- Literal ℓ: x or negation $\overline{x} = 1 - x$
- Pseudo-Boolean (PB) constraint: Integer linear inequality over literals

$$3x_1 + 2x_2 + 5\overline{x}_3 \geq 5$$

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$$3x_1 + 2x_2 + 5\overline{x}_3 = 5 \longrightarrow \begin{array}{l} 3x_1 + 2x_2 + 5\overline{x}_3 \geq 5 \\ 3x_1 + 2x_2 + 5\overline{x}_3 \leq 5 \end{array}$$

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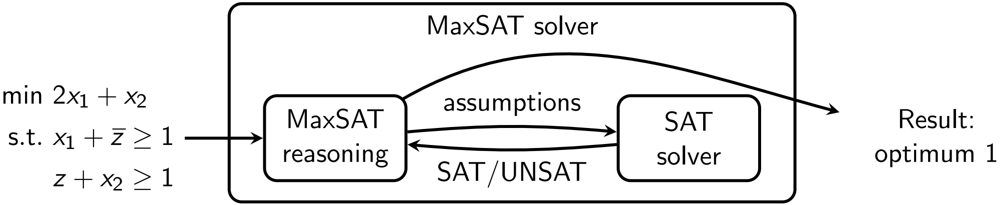
- Clause: Disjunction of literals or at-least-one constraint

$$x_1 \vee \bar{x}_2 \vee \bar{x}_3 \iff x_1 + \bar{x}_2 + \bar{x}_3 \geq 1$$

- CNF formula can be viewed as a collection of pseudo-Boolean constraints

OLL-Style Core-Guided MaxSAT Solving [MDM14]



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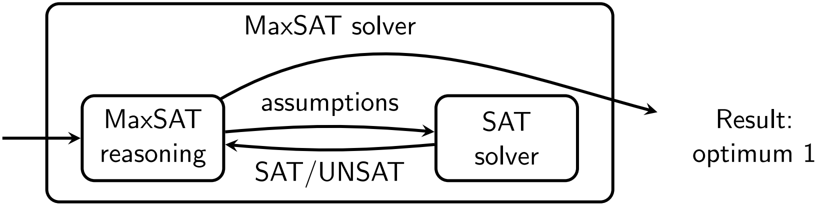

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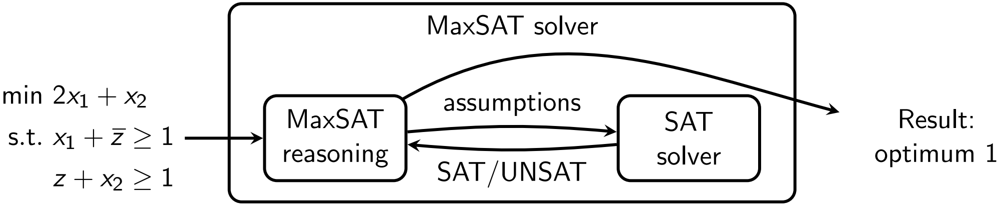

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3. Reformulate objective and goto 1.

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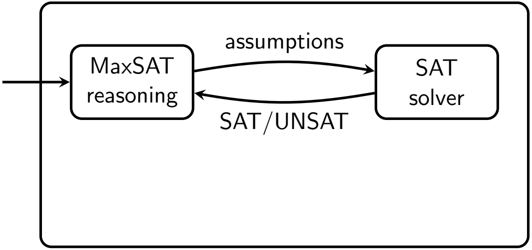

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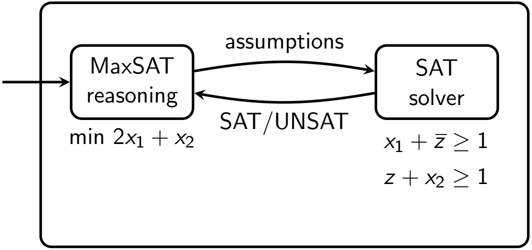

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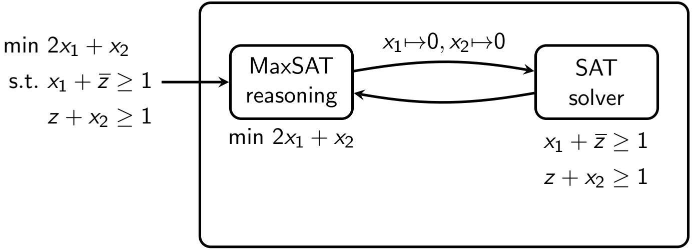

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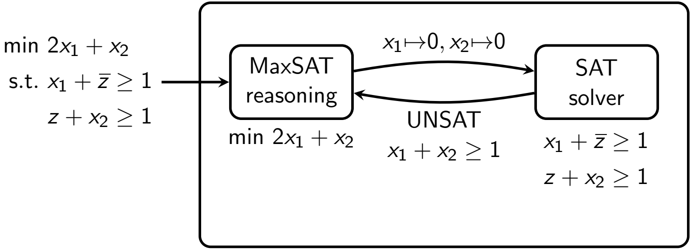

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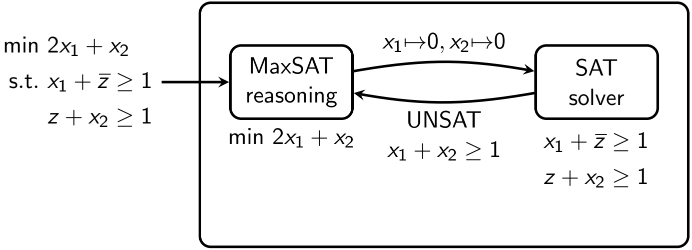

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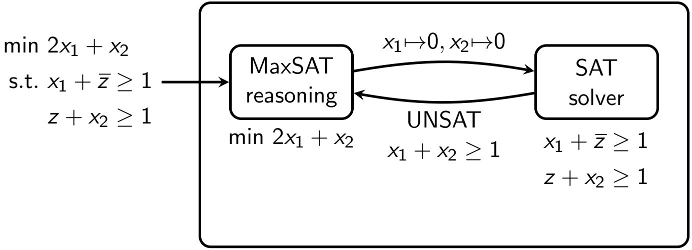

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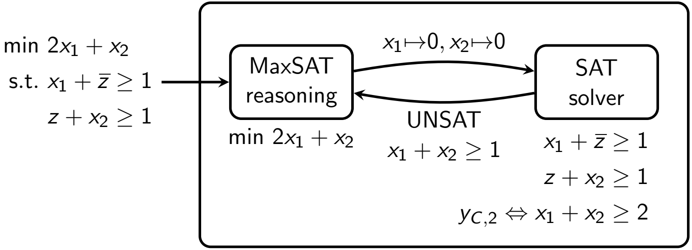

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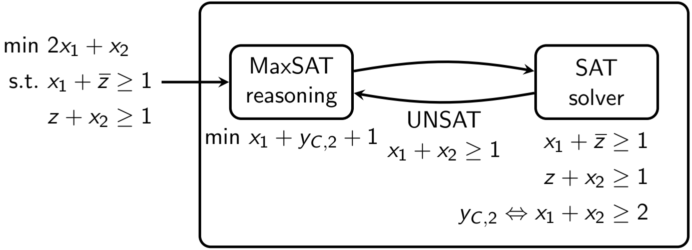

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- Definition of counter variables encoded to CNF using totalizers
- Using $x_1 + x_2 = 1 + y_{C,2}$, reformulate objective from $2x_1 + x_2$ to $x_1 + y_{C,2} + 1$

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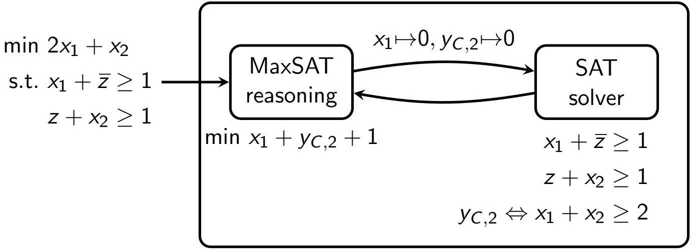

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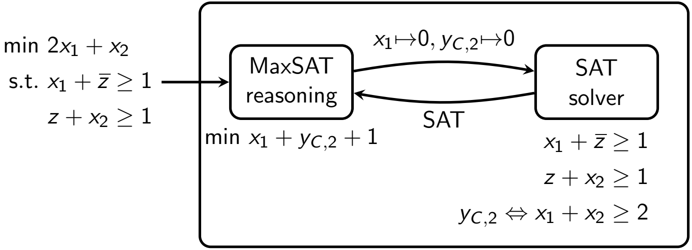

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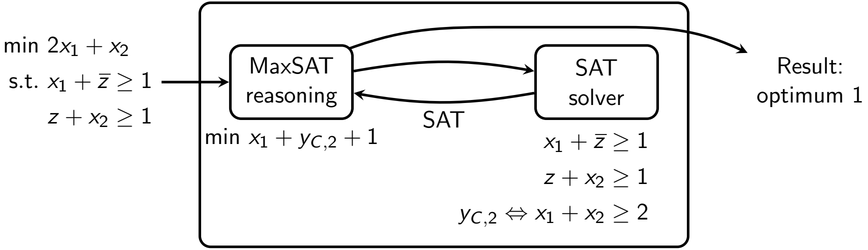

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- Best possible assumptions about objective satisfy all constraints
- Optimal solution found with value 1

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Rules:

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$$\frac{}{x \geq 0} \qquad \frac{}{\overline{x} \geq 0}$$

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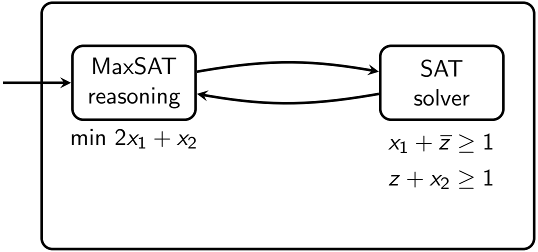

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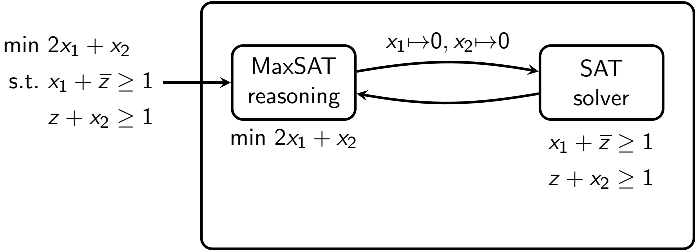

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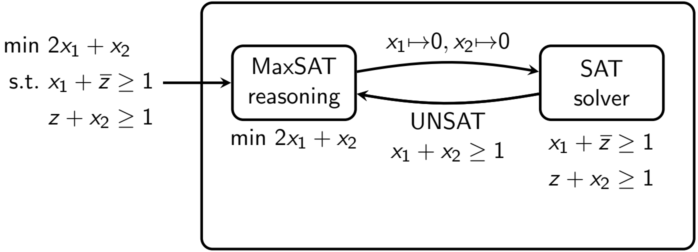

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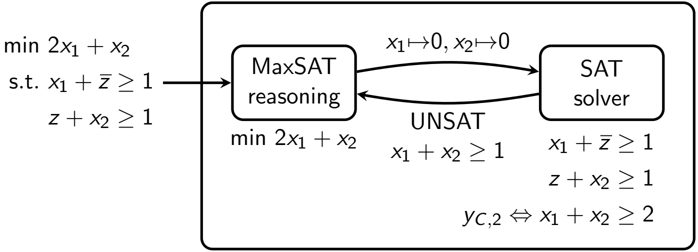

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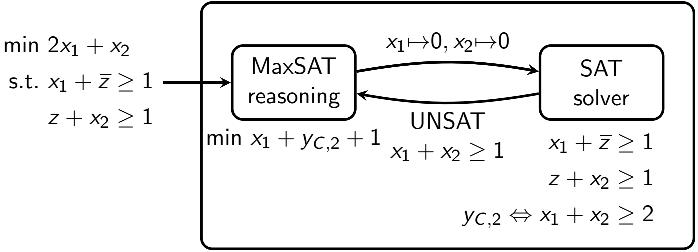

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- Core $C : x_1 + x_2 \geq 1$ is implied trivially by reverse unit propagation
- Introduce counter variables definition by reification
- Provide proof logging for totalizers leveraging [GMNO22, VDB22]
- Maintain invariant original objective equal to reformulated objective in proof
- This is $x_1 + x_2 = 1 + y_{C,2}$

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$$\Longrightarrow\ x_1 + x_2 \geq y_{C,1} + y_{C,2}\ \Longrightarrow\ x_1 + x_2 \geq 1 + y_{C,2}$$

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$$\dfrac{\dfrac{\dfrac{y_{C,1} \Rightarrow x_1 + x_2 \geq 1}{\overline{y}_{C,1} + x_1 + x_2 \geq 1}\text{ Reification} \quad \dfrac{y_{C,2} \Rightarrow x_1 + x_2 \geq 2}{2\overline{y}_{C,2} + x_1 + x_2 \geq 2}}{\overline{y}_{C,1} + 2\overline{y}_{C,2} + 2x_1 + 2x_2 \geq 3}\text{ Addition}}{\overline{y}_{C,1} + \overline{y}_{C,2} + x_1 + x_2 \geq 2}\text{ Divide by 2}$$

$$\implies x_1 + x_2 \geq y_{C,1} + y_{C,2} \implies x_1 + x_2 \geq 1 + y_{C,2}$$

$$\dfrac{\dfrac{\dfrac{y_{C,1} \Leftarrow x_1 + x_2 \geq 1}{2y_{C,1} + \overline{x}_1 + \overline{x}_2 \geq 2}\text{ Reification} \quad \dfrac{y_{C,2} \Leftarrow x_1 + x_2 \geq 2}{y_{C,2} + \overline{x}_1 + \overline{x}_2 \geq 1}}{2y_{C,1} + y_{C,2} + 2\overline{x}_1 + 2\overline{x}_2 \geq 3}\text{ Addition}}{y_{C,1} + y_{C,2} + \overline{x}_1 + \overline{x}_2 \geq 2}\text{ Divide by 2}$$

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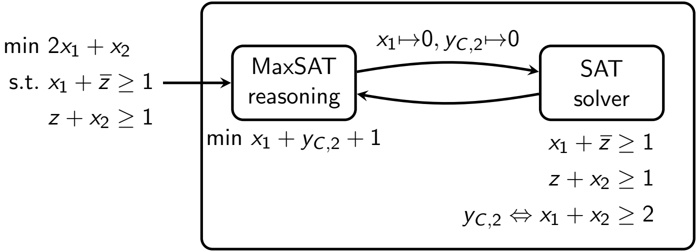
Example: Proof Logging for Core-Guided MaxSAT Solving (2/2)



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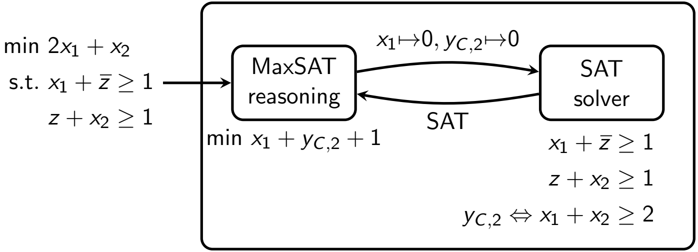

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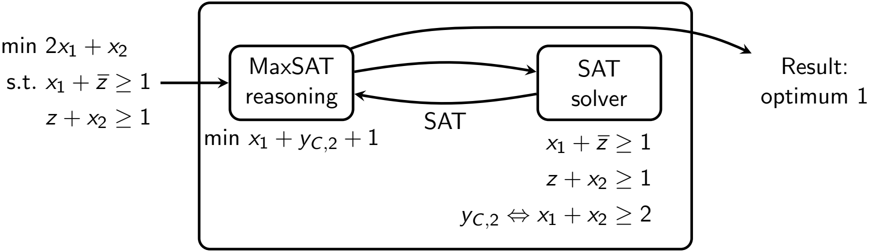

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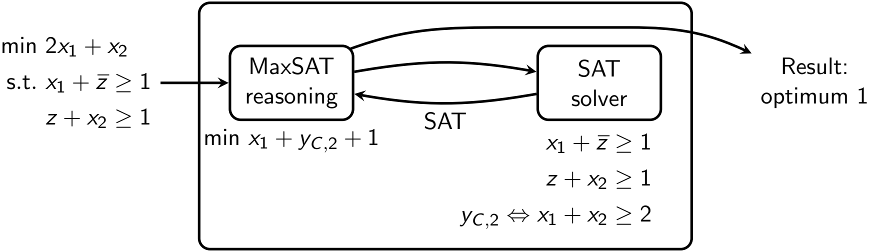

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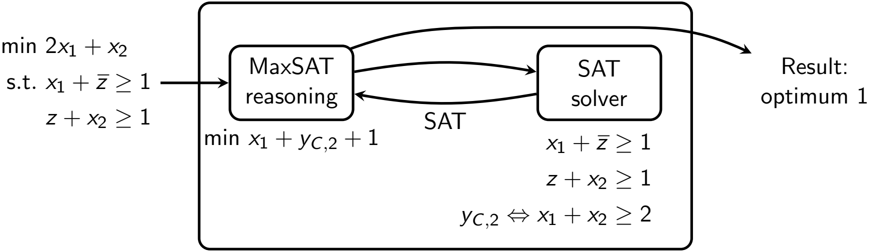

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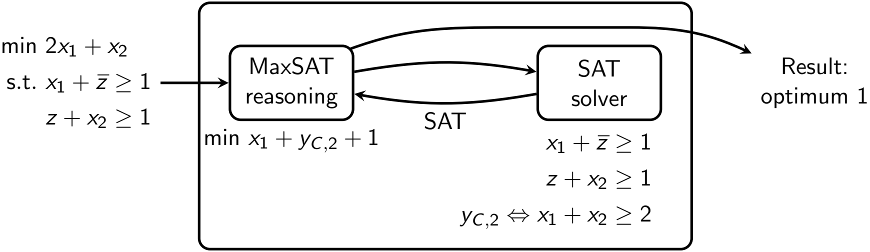

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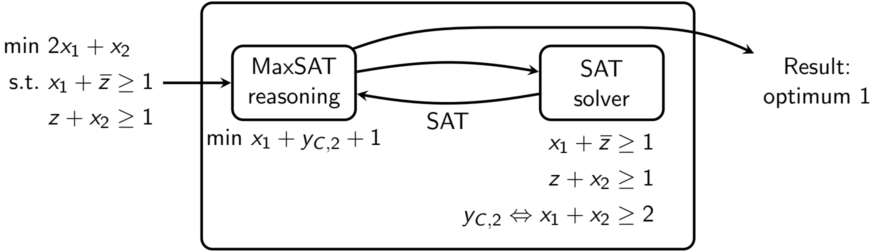

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- Contradicts assumption
- Solution must be optimal

General Approach: Proof Logging for Core-Guided MaxSAT Solving

1. Record solution in proof to establish upper bound UB
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- For anytime solving: Guarantee on lower and upper bound without step 3

Advanced Techniques for Core-Guided MaxSAT

- Important to deal with all state-of-the-art solver techniques
- Additional techniques that are skipped in this talk
  - Intrinsic at-most-one constraints [IMM19]
  - Hardening [ABGL12]
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- Very convenient to do in our proof format → see our paper [BBN+23]

Experimental Evaluation

- Implemented certifying version of state-of-the-art solver CGSS[1] [IBJ21]
- Proof checked with proof checker VeriPB[2]
- Benchmarks from MaxSAT Evaluation 2022[3]
  - 607 unweighted instances and 594 weighted instances

[1] https://gitlab.com/MIAOresearch/software/certified-cgss
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First result:

- Discovered bugs in CGSS (and also RC2, on which CGSS is based)
  - All claimed optimal solutions correct for our benchmarks set
  - But solver reasoning sometimes wrong
  - Solver bug could lead to erroneous claims of optimality for other benchmarks

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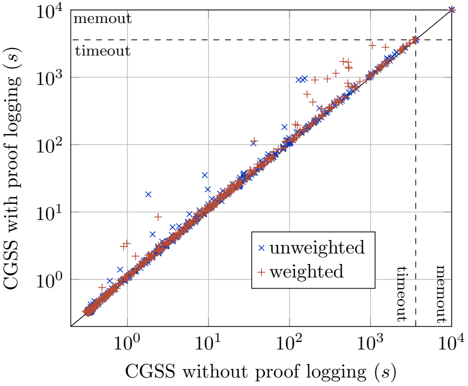
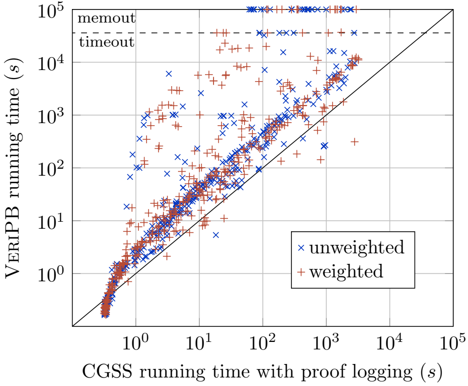
Experimental Results



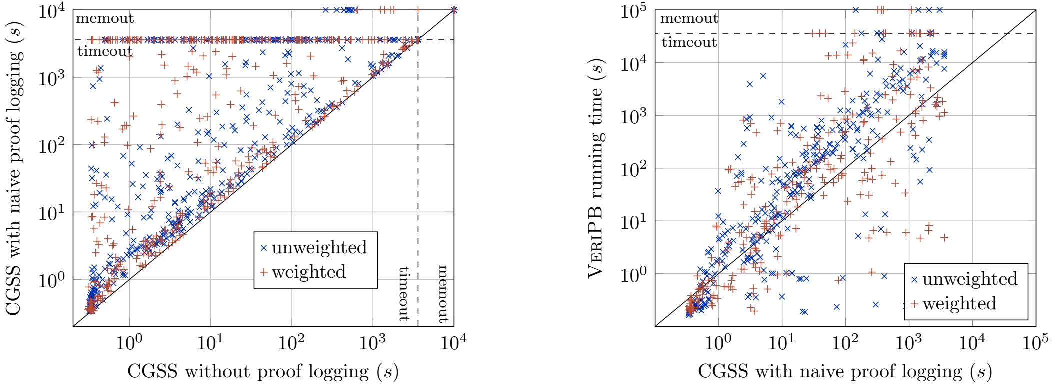
(a) Overhead for proof logging.



(b) Solving versus checking.

- Low proof logging overhead (8.8% median)
- Checking time could be improved (VeriPB not optimized for SAT solver proofs)

How about Using a SAT Solver to Certify Result?



(a) Overhead for proof logging. (b) Solving versus checking.

- Encode objective-improving constraint to CNF and solve with SAT solver (Kissat)

Future Work

Further proof logging:

- State-of-the-art linear SAT-UNSAT search solver (like Pacose)
- Implicit hitting set MaxSAT solver
  - Fundamental challenge: proof logging for MIP solver
- Pseudo-Boolean optimization

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Improving performance and reliability:

- Optimize VeriPB for SAT solver proofs
- Backwards checking/trimming for verification (as in DRAT-trim [HHW13a])
- Formally verified proof checker [BMM+23]

The Sales Pitch For Proof Logging

1. Certifies correctness of computed results
2. Detects errors even if due to compiler bugs, hardware failures, or cosmic rays
3. Debugging support during development [EG21, GMM+20, KM21, BBN+23]
4. Facilitates performance analysis
5. Helps identify potential for further improvements
6. Enables auditability
7. Serves as stepping stone towards explainability

Conclusion

- MaxSAT: successful optimization paradigm, but without proof logging
- Pseudo-Boolean reasoning supports MaxSAT proof logging
- This work: Proof logging for state-of-the-art core-guided MaxSAT solving
- Hopefully step towards general adoption of proof logging for MaxSAT

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Pseudo-Boolean reasoning provides unified proof logging method for:

- SAT solving (including advanced techniques) [GN21, BGMN22]
- Constraint programming [EGMN20, GMN22]
- Graph problems [GMN20, GMM+20]
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Thank you for your attention!

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