An Empirical Study of the Effect of Learnt Clause on the Structural Measures of SAT problems

Pragmatics of SAT, July 4th 2023, Alghero, Italy

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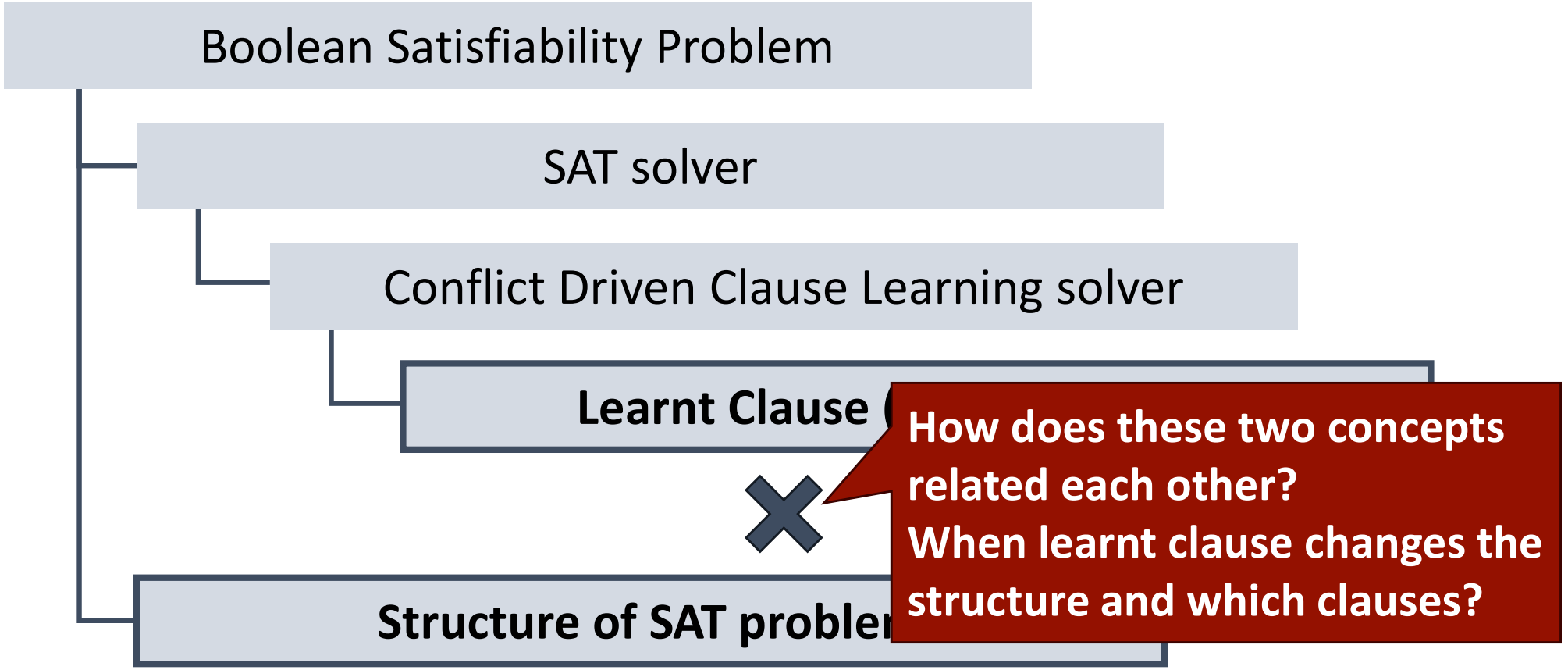
Today's topic



Clause learning and Literal Block Distance

Clause Learning

- A technique to improve the efficiency of solver by preventing the same contradictions. The result of learning is saved as a learnt clause
- Learning system contains 'clause management system' to maintain appropriate number of clauses through deletion of the worse clause

Literal Block Distance[1]

- A widely used metric to evaluate the quality of learnt clause
- Given a clause $\boldsymbol{c}$, its LBD is calculated as $\boldsymbol{LBD(c) = |\{d(l): l \in c\}|}$ where $\boldsymbol{d(l)}$ is the decision level of literal $\boldsymbol{l}$, and $\boldsymbol{|X|}$ indicates the number of elements in the set $\boldsymbol{X}$

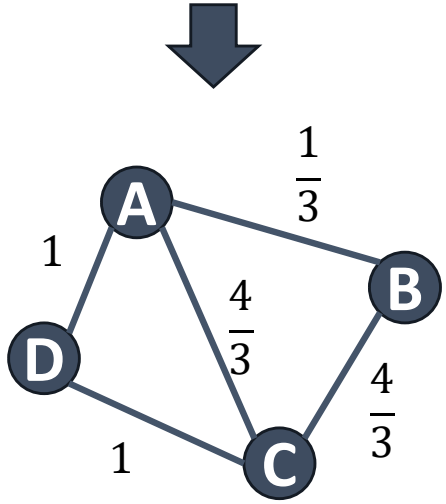
1. G. Audemard and L. Simon, "Predicting learnt clauses quality in modern SAT solvers," In IJCAI (2009)

Conjunctive Normal Form and Graph Representation

CNF (Conjunctive Normal Form)

- A logical structure where an expression is represented as an AND of clauses, with each clause being an OR of variables or their negations
- A standardized format for the input of SAT solver

$$(A \vee B \vee \neg C) \wedge (\neg A \vee D) \wedge (B \vee \neg C) \wedge (\neg A \vee C) \wedge (\neg C \vee D)$$



Graph representation of SAT

- VIG (Variable Incident Graph) is a graph $\boldsymbol{G(\psi)}$ for an instance $\boldsymbol{\psi}$ with variables $\boldsymbol{V}$ and clauses $\boldsymbol{C}$, $\boldsymbol{V}$ denotes the nodes of VIG $\boldsymbol{G}$ and edges $\boldsymbol{E}$ denote a clause $\mathbf{c} \in \boldsymbol{C}$ exists containing variables $\boldsymbol{v_i}, \boldsymbol{v_j} \in \boldsymbol{V}$
- The weight of edge $w\left(e_{\boldsymbol{v_i},\boldsymbol{v_j}}\right) \coloneqq \sum_{\mathbf{c}\in\boldsymbol{C},\boldsymbol{v_i},\boldsymbol{v_j}\in\boldsymbol{V}} \frac{1}{\binom{|c|}{2}}$

Structural measures: treewidth and modularity

Treewidth

- Treewidth := $\boldsymbol{min_T max_{i \in V(T)} |B_i| - 1}$, indicating how the graph is like a tree. Here, $\boldsymbol{T = (V(T), E(T))}$ is tree decomposition of $\boldsymbol{g}$, and $\boldsymbol{B_i \in V}$ represents a bag

Community

- Community is the group of nodes that have a high degree of interconnections
- We used Louvain community detection algorithm[1] to detect the community

Modularity[2]

- $\boldsymbol{G = (V, E)}$ and its community partition $\boldsymbol{P = \{p_1, p_2, \dots, p_k\}}$ of $\boldsymbol{V}$ where $\boldsymbol{P}$ is pairwise disjoint and $\boldsymbol{V = \sum_{i=1}^{k} p_i}$
- Modularity $\boldsymbol{Q(G, P) := \frac{1}{2m} \sum_{i,j}^{|V|} \left[A_{ij} - \frac{k(i), k(j)}{2m} \right] \delta(c_i, c_j)}$ where $\boldsymbol{A_{ij}}$ is the adjacency matrix $\boldsymbol{A}$ at $\boldsymbol{i}$ and $\boldsymbol{j}$, $\boldsymbol{k(i)}$ is the sum of the edge weight of vertices $\boldsymbol{i}$, $\boldsymbol{m}$ is $\boldsymbol{\sum_i k(i) / 2}$, $\boldsymbol{c_i \in P}$ is the community of $\boldsymbol{i}$, $\boldsymbol{\delta(c_i, c_j)}$ equals 1 if $\boldsymbol{c_i = c_j}$, otherwise 0

1. Blondel, V.D., et.al., Fast unfolding of communities in large networks. Journal of Statistical Mechanics: Theory and Experiment (2008)
2. Newman, M.E.J., Girvan, M.: Finding and evaluating community structure in networks. Phys. Rev. E 69(2), 026113 (2004)

Related work and Our goal

Related work

- Ansótegui, C et.al., (2015)[1] showed that modularity Q decreases through the learning and that most of clauses decrease the Q while only some increase it
- Vallade, V et.al., (2020)[2] showed that there is a sort of correlation between the LBD and the number of community. However, they also stated that using the number of community as an alternative of LBD is misleading

Our goal

- Deepen our understanding of how the learnt clauses changes the structure, measured by the value of modularity and treewidth
- Reveal the correlation between the quality of clause (by LBD) and their impact on the structure (by the delta of modularity by adding the learnt clause)

1 Ansótegui, C et.al., Using Community Structure to Detect Relevant Learnt Clauses, in SAT (2015)
2 Vallade, V et.al., Community and LBD-Based Clause Sharing Policy for Parallel SAT Solving, in SAT (2020)

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Experimental setup

Objective

- See how and when treewidth and modularity changes from the time $\boldsymbol{org}$ and $\boldsymbol{last}$ of search where $\boldsymbol{org}$ is time before preprocessing and $\boldsymbol{last}$ is time before the terminating

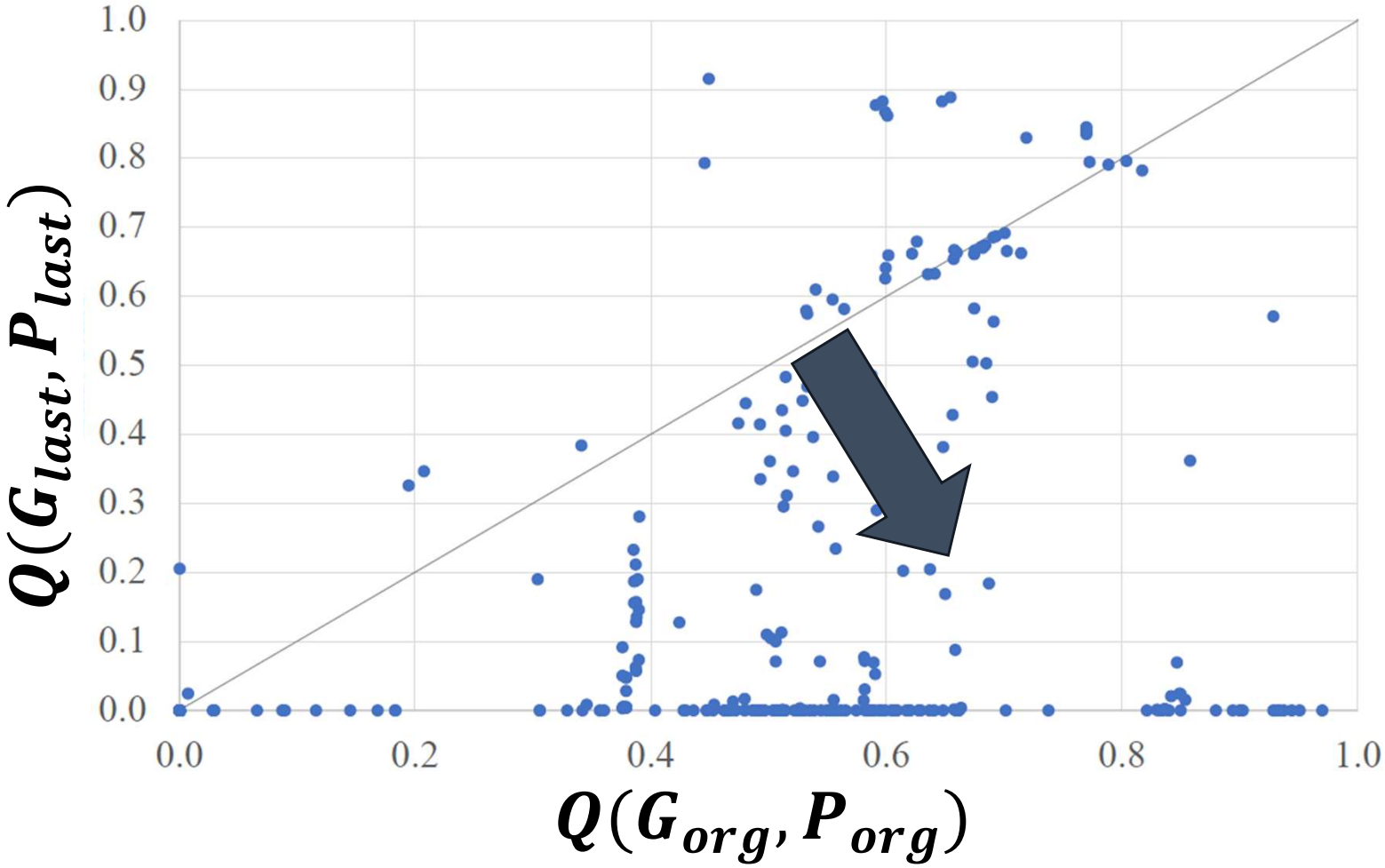
Definition

- For time $\boldsymbol{t}$ during search, $\boldsymbol{C_t}$ is the learnt clauses maintained by solver at time $\boldsymbol{t}$, we define $\boldsymbol{\psi_t = \psi_{org} + C_t}$ and $\boldsymbol{G_t = G(\psi_t)}$ is the VIG of $\boldsymbol{\psi_t}$

Experimental setup

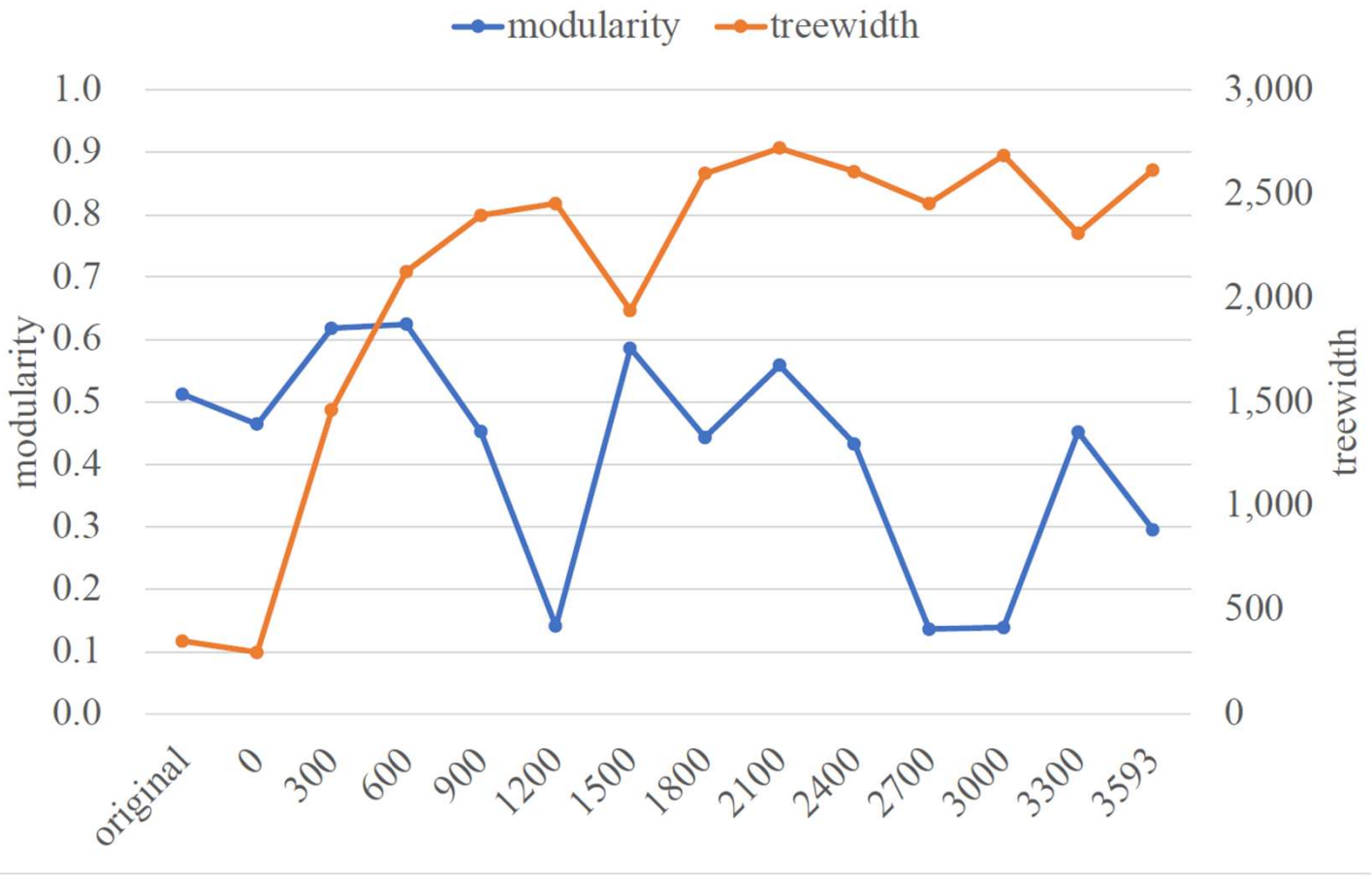
- Glucose 4.2.1 as the base solver with timeout at 3600 s
- Benchmark 400 instances from the SAT competitions 2021 main track
- FlowCutter to calculate upper-bound of treewidth with 20min time out and python networkx community package for Modularity calculation within 24 h
- Computer on AMD 3995WX and 512 GB RAM (128 GB x 4, DDR4-3200 MHz)

Modularity change from *org* to *last*

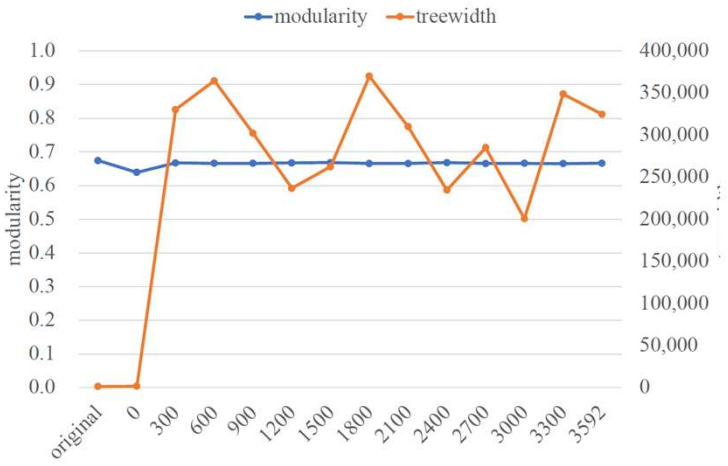


When learnt clause changes the structure?

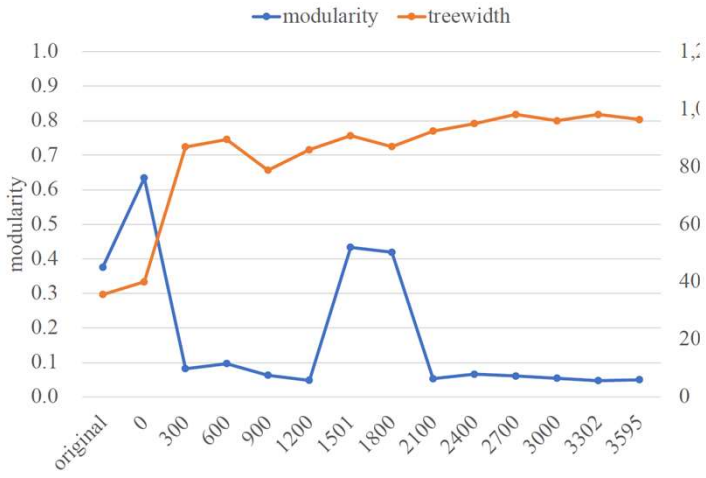
trend-sum_of_3_cubes_87_bits_75.cnf



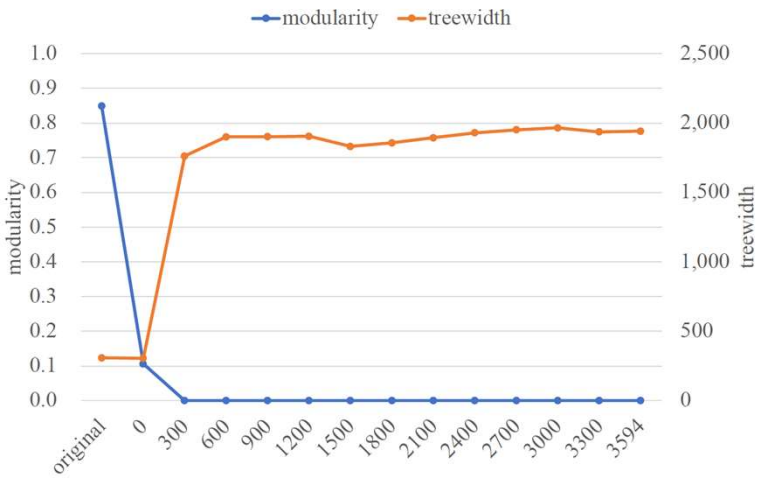
When learnt clause changes the structure?



trend-spg_300_300.cnf



trend-puzzle37_unsat.cnf



trend-ktf_TF-3.tf_2_0.02_24.cnf

Results of all instances are available in
https://github.com/messhiida/PoS2023-images.git

Experimental setup

Objective

- Compares the value of $\boldsymbol{\Delta Q(c)}$ and LBD value of learnt clause $\boldsymbol{c}$

Definition

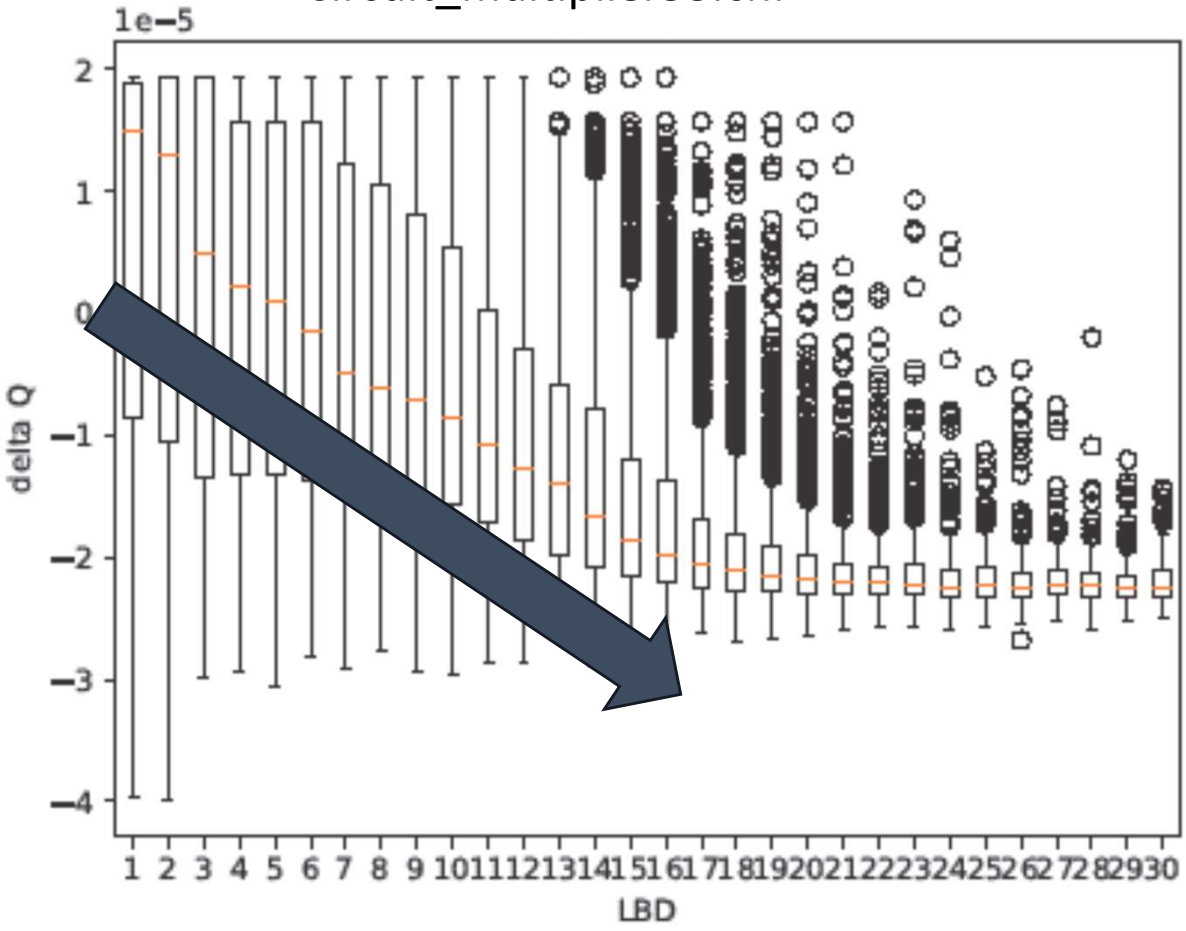
- For a learnt clause $\boldsymbol{c}$, $\boldsymbol{\Delta Q(c) := Q\big(G\big(\psi_{org} + c\big), P'\big) - Q\big(G\big(\psi_{org}\big), P\big)}$, assuming $P' = P$

Experimental setup

- Ignored the clauses of size 100+ or LBD 30+
- Glucose 4.2.1 as the base solver with timeout at 3600 s
- Benchmark 400 instances from the SAT competitions 2021 main track
- Modularity calculation within 24 h by networkx community package
- Computer equipped with an AMD Threadripper Pro 3995WX processor (64 cores) and 512 GB (128 GB 4 slots, DDR4-3200 MHz) of RAM

Which learnt clause changes the structure? (1/2)

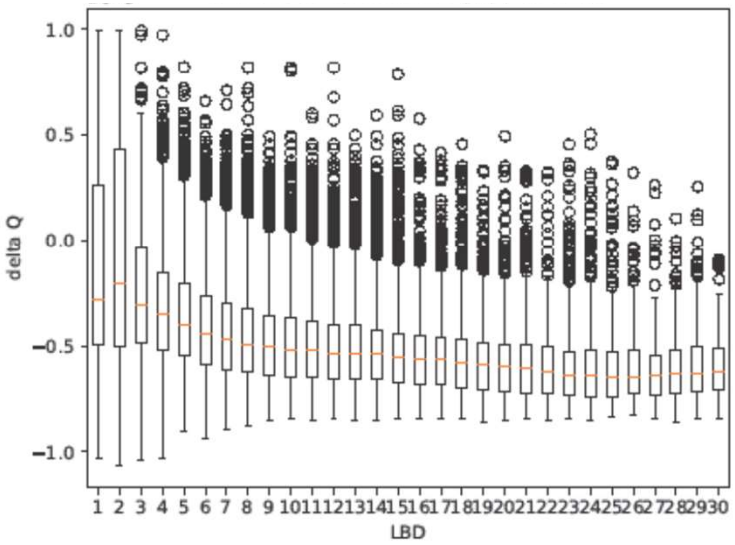
Circuit_multiplier53.cnf



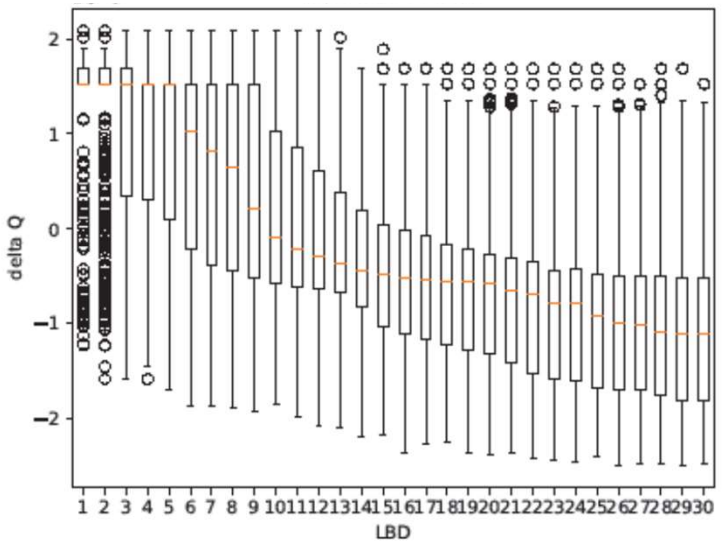
- Larger delta Q for smaller LBD in average and distribution
- Larger LBD clauses tends to decrease the modularity Q, meaning “destroy the structure”
- The impact of size is unclear

Which learnt clause changes the structure? (1/2)

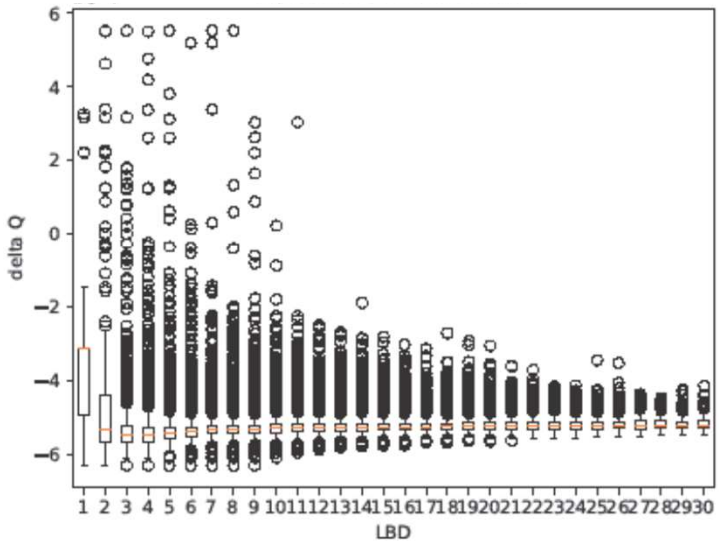
at-least-two-maris-s03-gripper11.cnf



mp1-Nb7T44.cnf



Mycielski-10-hints-10.cnf



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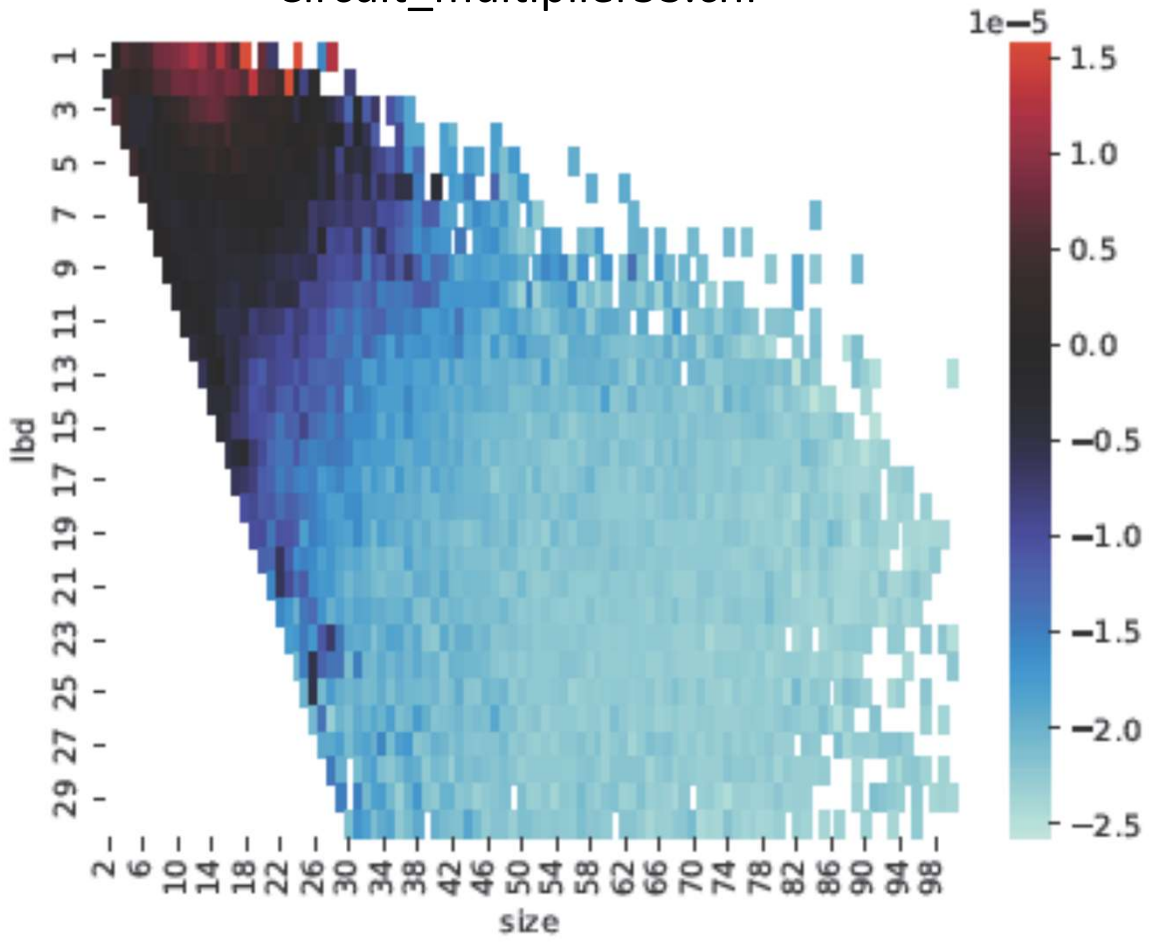
Which learnt clause changes the structure? (2/2)

Circuit_multiplier53.cnf

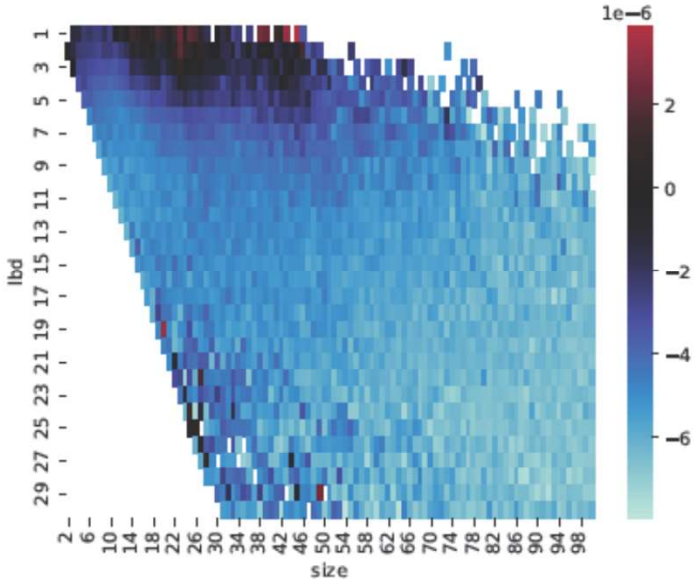


- Larger delta Q for smaller size
- Larger delta Q for smaller LBD
- In the same size, lager delta Q for smaller LBD (in same size)

- Larger LBD clauses tends to decrease the modularity Q, meaning “destroy the structure” even in same size

Which learnt clause changes the structure? (2/2)

at-least-two-maris-s03-gripper11.cnf



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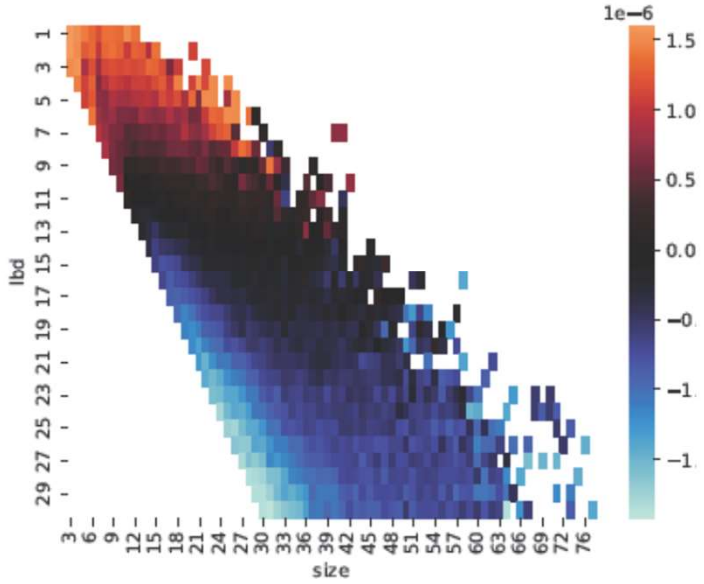
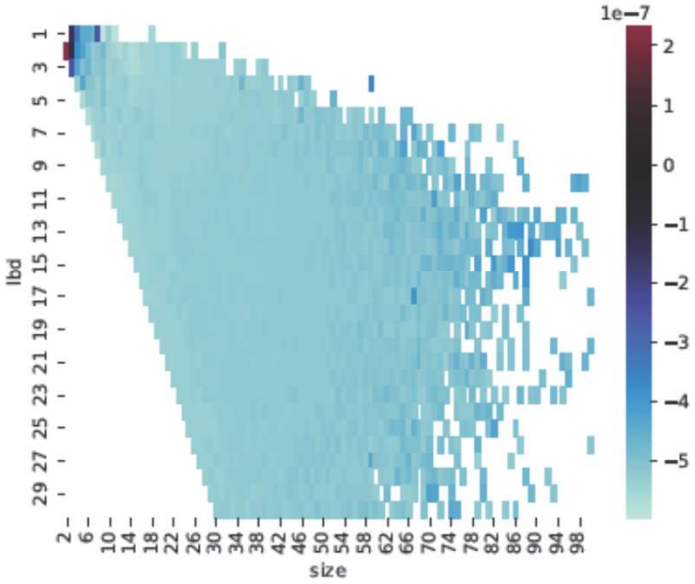


Mycielski-10-hints-10.cnf



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Differences of all clauses and positive delta Q clauses

	Positive $\Delta Q(c)$	All clauses
Ratio	24%	(100%)
Average LBD	6.75	11.93
Number of Community	3.04	6.90

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Discussion

When and how much learnt clause changes the structure?

- We observed that the structure changes immediately after search start and then it shows zig-zag trend
- This is assumed as the result of the deletion of learnt clauses, and the deletion of worse evaluated clauses with larger LBD values turns back the changed structure

Which learnt clause changes the structure?

- The better clauses decrease less or even increase the modularity value than worse clauses.
- Positive delta Q clauses had lower LBD values and fewer # community
- Idea of the learnt clause evaluation method using the delta Q

Thank you

ありがとうございました

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